

sample becomes

$$Y_m = K_1 \left(\sigma_{xx} + j\omega\epsilon_{xx} + \frac{\sigma_{12}^2}{\sigma_{yy} + j\omega\epsilon_{yy}} \right). \quad (35)$$

On the other hand, if the material is isotropic, the expression for the equivalent admittance of the sample becomes

$$(Y_m)_{iso} = K_1(\sigma + j\omega\epsilon). \quad (36)$$

REFERENCES

- [1] K. S. Champlin, J. D. Holm, and G. H. Glover, "Electrodeless determination of semiconductor conductivity from TE_{01} -mode reflectivity," *J. Appl. Phys.*, vol. 38, pp. 96-98, Jan. 1967.
- [2] F. A. D'Altroy and H. Y. Fan, "Microwave transmission in p-type germanium," *Phys. Rev.*, vol. 94, pp. 1415-1416, June 1954.
- [3] H. Jacobs, F. A. Brand, J. D. Meindl, S. Weitz, and R. Benjamin, "New microwave technique in the measurement of semiconductor phenomenon," in *IRE Int. Conv. Rec.*, vol. 10, pt. 3, pp. 30-42, Mar. 1962.
- [4] A. N. Datta and B. R. Nag, "Techniques for the measurement of complex microwave conductivity and the associated errors," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 162-166, Mar. 1970.
- [5] K. S. Champlin and G. H. Glover, "'Gap effect' in measurement of large permittivities," *IEEE Trans. Microwave Theory Tech. (Corresp.)*, vol. MTT-14, pp. 397-398, Aug. 1966.
- [6] J. D. Holm, "Microwave conductivity of silicon and germanium," Ph.D. dissertation, the University of Minnesota, Minneapolis, 1967.
- [7] H. M. Altschuler, "Dielectric constant," in *Handbook of Microwave Measurements*, vol. II, M. Sucher and J. Fox, Eds. Brooklyn, N. Y.: Polytechnic Press, 1963, pp. 495-590.
- [8] H. R. G. Casimir, "On the theory of electromagnetic waves in resonant cavities," *Philips Res. Reps.*, vol. 6, 1951.
- [9] E. L. Ginzton, *Microwave Measurements*. New York: McGraw-Hill, 1957.
- [10] R. A. Waldron, "Perturbation theory of resonant cavities," *Proc. Inst. Elec. Eng.*, vol. 107C, pp. 272-274, Sept. 1960.
- [11] I. I. Eldumiati, "Bulk semiconductor materials for millimeter- and submillimeter-wave detection," Electron Physics Laboratory, the University of Michigan, Ann Arbor, Tech. Rep. 118, Grant NGL 23-005-183, Dec. 1970.
- [12] S. R. De Groot and P. Mazur, "Extension of Onsager's theory of reciprocal relations," *Phys. Rev.*, vol. 94, pp. 218-226, Apr. 1954.

Traveling-Wave Coherent Light-Phase Modulator

M. EZZAT EL-SHANDWILY, MEMBER, IEEE, AND SAID M. EL-DINARY

Abstract—The use of a rectangular waveguide partially loaded with electrooptic material as a laser beam phase modulator is analyzed theoretically. The characteristic equation, fields, power, and attenuation are obtained in terms of normalized parameters. Design procedure of the modulator is given with particular reference to KDP.

I. INTRODUCTION

THE USE OF electrooptic (EO) materials for amplitude or phase modulation of a coherent light beam has been analyzed by several authors [1]. For a traveling-wave modulator, a waveguide partially filled with the EO crystal is usually used to support the modulating signal. Kaminow and Liu [2] analyzed the propagation characteristics of a parallel plate guide partially loaded with KDP crystal. In their analysis, TEM fields are used to produce phase modulation of the coherent beam. They found that for practical KDP crystal dimensions, 3 GHz is the highest frequency for broad-band operation for the collinear geometry. Higher order modes have not been considered. Peters [3] demonstrated experimentally the operation of a traveling-wave coherent light-phase modulator. The

structure used was similar to that analyzed theoretically by Kaminow and Liu [2]. A modulation index of unity was obtained with a modulating power of 12 W. Chen and Lee [4] analyzed the propagation characteristics of a cylindrical waveguide partially filled with a cylindrical dielectric light modulation material. They considered only modes with no angular variation. Their analysis reveals that for TM mode propagation there is a nondispersive region which is suitable for broad-band modulation.

Recently, Putz [5] described an experimental microwave-light modulator, using a ring-phase traveling-wave circuit, with KDP crystals filling the space inside the rings. The KDP crystal is used in the longitudinal mode, i.e., with the light beam along the optic axis. The modulator uses 10 W of input power to produce AM modulation at a modulation depth of 40 percent with 10-percent bandwidth. However, for phase modulation it gives only a modulation index of 0.2 rad. Vartanian *et al.* [6] investigated the propagation characteristics of TE_{n0} modes in dielectric loaded rectangular waveguide. Our work is concerned with the same structure for use as a laser beam phase modulator.

In Section II, we give a brief description of the phase change in a laser beam traveling through an EO crystal in the presence of an RF field. Section III treats the problem of electromagnetic wave propagation through

Manuscript received February 1, 1971; revised March 15, 1971.

M. E. El-Shandwily is with the National Research Center, Electrical and Electronic Research Laboratory, Sh. El-Tahrir, Dokki-Cairo, Egypt.

S. M. El-Dinary is with the Atomic Energy Establishment, Cairo, Egypt.

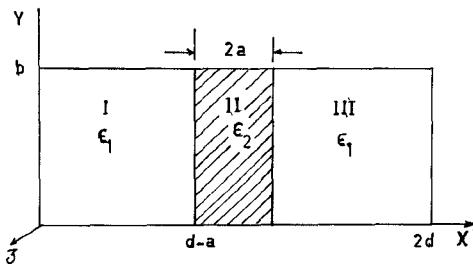


Fig. 1. Model used for analysis.

a rectangular waveguide partially loaded with an EO crystal. The design of the modulator is treated in Section IV.

II. PHASE MODULATOR OPERATION

The modulator considered in this paper is a rectangular waveguide filled with two different dielectric media, as shown in Fig. 1. Medium II is the EO crystal which is placed symmetrically at the center. The modulator uses the KDP or ADP crystal in the r_{63} transverse mode [7].

Consider a crystal with principal axes x_1 , x_2 , and x_3 . When a field E_3 is applied along the x_3 axis, the principal axes become x_3 , x_1' , and x_2' at 45° to the x_1 and x_2 axes. The refractive index for x_3 remains n_3 while

$$n_{x_1'} = n_1 - 1/2n_1^3r_{63}E_3$$

$$n_{x_2'} = n_1 + 1/2n_1^3r_{63}E_3.$$

When a coherent light beam polarized along x_2' passes through the EO crystal parallel to the x_1' axis, it will experience a phase modulation. Since the $x_3=0$ section of the index ellipsoid is circular, it follows that in the absence of an applied electric field an x_1' directed light beam polarized in the $x_3=0$ plane will propagate as $\exp j\omega(t-(n_1/c)x_1')$. After traversing a distance L , the emerging beam will experience a phase retardation with respect to the entering beam equal to

$$\phi = - \int_0^L \frac{\omega n_1}{c} dx_1'. \quad (1)$$

When the electric field E_3 is applied the phase retardation (1) becomes

$$\phi' = - \int_0^L \frac{\omega}{c} (n_1 - 1/2n_1^3r_{63}E_3) dx_1'. \quad (2)$$

The change in phase due to the electrooptic effect is

$$\Delta\phi = - \frac{\pi}{\lambda_0} n_1^3 r_{63} \int_0^L E_3 dx_1'$$

where λ_0 is the free space wavelength. When E_3 travels with the same velocity as the light beam, then, taking the attenuation into account, $\Delta\phi$ will be given by

$$\Delta\phi = - \frac{\pi}{\lambda_0} n_1^3 r_{63} E_3' \frac{(1 - e^{-\alpha L})}{\alpha} \quad (3)$$

where E_3' is the amplitude of the electric field and α is the attenuation constant.

When there is a velocity difference between the laser beam and the microwave field, the change in phase for the lossless case will be given by [2]

$$\Delta\phi = - \frac{\pi}{\lambda_0} n_1^3 r_{63} E_3' L \frac{\sin u}{u} \quad (4)$$

where

$$u = \frac{\omega L}{2} \left(\frac{1}{U} - \frac{1}{v} \right),$$

U velocity of light in the EO medium,

v phase velocity of the RF modulating field.

The EO crystal is placed in the waveguide such that its axes x_2' , x_3 , and x_1' coincide, respectively, with the x , y , and z axes for the waveguide.

III. THEORETICAL ANALYSIS OF THE STRUCTURE

In this section, the electromagnetic wave propagation along the structure shown in Fig. 1 will be analyzed. In general, the normal modes of propagation are neither TE nor TM, but a combination of both of them. However, when there is no variation of the electric field in the y direction, it will be possible to have either the TE or TM mode alone.

Consider the propagation of TE_{n0} modes; the propagation equation for H_z can be written in the following forms:

$$a) \frac{d^2 H_z}{dx^2} = -(\beta^2 \epsilon_1 - k^2) H_z, \quad 0 < k < \sqrt{\epsilon_1} \quad (5)$$

$$b) \frac{d^2 H_z}{dx^2} = 0, \quad k = \beta \sqrt{\epsilon_1} \quad \left. \right\} \text{in region I and III} \quad (6)$$

$$c) \frac{d^2 H_z}{dx^2} = (k^2 - \beta^2 \epsilon_1) H_z, \quad \beta \sqrt{\epsilon_1} < k < \beta \sqrt{\epsilon_2} \quad (7)$$

and

$$\frac{d^2 H_z}{dx^2} = -(\beta^2 \epsilon_2 - k^2) H_z, \quad \text{in region II} \quad (8)$$

where

$$\beta = \omega \sqrt{\mu_0 \epsilon_0} = \omega/c \quad \text{free-space propagation constant,} \\ \epsilon_1, \epsilon_2 \quad \text{the relative dielectric constants.}$$

The fields are assumed to vary as $\exp j(\omega t - kz)$ and the boundary conditions are the following:

$$E_{\tan} = E_y \text{ is continuous} \quad H_{\tan} = H_z \text{ is continuous.} \quad (9)$$

We introduce the normalized parameters

$$\nu = \beta a \epsilon_2^{1/2} = 2\pi \epsilon_2^{1/2} \frac{a}{\lambda_0}$$

proportional to the width of electrooptic medium measured in wavelength in that medium.

$$Q = \frac{\epsilon_1}{\epsilon_2} \leq 1, \quad \text{ratio of dielectric constants}$$

$$\xi = \left(\frac{d}{a} - 1 \right), \quad \text{ratio of cross sectional area of} \\ \text{media I and II} \quad (10)$$

$$m = \frac{k}{\beta \sqrt{\epsilon_2}} = \frac{c}{\nu \sqrt{\epsilon_2}}$$

where ν is the velocity of propagation of the electromagnetic wave. Solving (5)–(9) and using Maxwell's equations, the expressions for the electric fields in the different regions in terms of the normalized parameters, (10), are as follows.

Case a: $(0 < m < \sqrt{Q})$

$$E_{1y} = E_0 \sin \left(\nu \sqrt{Q - m^2} \frac{x}{a} \right) \quad (11)$$

$$E_{2y} = E_0 \left[\sin \left(\nu \xi \sqrt{Q - m^2} \right) \cos \left(\nu \sqrt{1 - m^2} \left(\xi - \frac{x}{a} \right) \right) \right. \\ \left. - \sqrt{\frac{Q - m^2}{1 - m^2}} \cos \left(\nu \xi \sqrt{Q - m^2} \right) \right. \\ \left. \cdot \sin \left(\nu \sqrt{1 - m^2} \left(\xi - \frac{x}{a} \right) \right) \right] \quad (12)$$

$$E_{3y} = - E_0 \left[\cos \left(2\nu \sqrt{1 - m^2} \right) - \sqrt{\frac{1 - m^2}{Q - m^2}} \right. \\ \left. \cdot \tan \left(\nu \xi \sqrt{Q - m^2} \right) \sin \left(2\nu \sqrt{1 - m^2} \right) \right] \\ \cdot \sin \left(\nu \sqrt{Q - m^2} \left(2\xi + 2 - \frac{x}{a} \right) \right) \quad (13)$$

and

$$H_x = - \frac{1}{Z_0} E_y \quad (14)$$

where $Z_0 = \omega \mu / k$ is the wave impedance, μ is the permeability, and E_0 is a constant. m can be determined from

the characteristic equation:

$$(Q - m^2) \cos^2 \left(\xi \nu \sqrt{Q - m^2} \right) \sin \left(2\nu \sqrt{1 - m^2} \right) \\ - (1 - m^2) \sin^2 \left(\xi \nu \sqrt{Q - m^2} \right) \cdot \sin \left(2\nu \sqrt{1 - m^2} \right) \\ + 2\sqrt{(Q - m^2)(1 - m^2)} \cos \left(\xi \nu \sqrt{Q - m^2} \right) \\ \cdot \sin \left(\xi \nu \sqrt{Q - m^2} \right) \cos \left(2\nu \sqrt{1 - m^2} \right) = 0. \quad (15)$$

Case b: $(m = \sqrt{Q})$

$$E_{1y} = E_0 \left(\frac{x}{a} \right) \quad (16)$$

$$E_{2y} = - \frac{E_0}{\nu \sqrt{1 - Q}} \left[\sin \left(\nu \sqrt{1 - Q} \left(\xi - \frac{x}{a} \right) \right) \right. \\ \left. - \nu \xi \sqrt{1 - Q} \cos \left(\nu \sqrt{1 - Q} \left(\xi - \frac{x}{a} \right) \right) \right] \quad (17)$$

$$E_{3y} = - E_0 \left[\cos \left(2\nu \sqrt{1 - Q} \right) - \nu \xi \sqrt{1 - Q} \right. \\ \left. \cdot \sin \left(2\nu \sqrt{1 - Q} \right) \left(2\xi + 2 - \frac{x}{a} \right) \right] \quad (18)$$

and

$$H_x = - \frac{1}{Z_0} E_y \quad (19)$$

where ν can be determined from the following equation:

$$[\nu^2 \xi^2 (1 - Q) - 1] \sin \left(2\nu \sqrt{1 - Q} \right) \\ - 2\nu \xi \sqrt{1 - Q} \cos \left(2\nu \sqrt{1 - Q} \right) = 0. \quad (20)$$

Case c: $(\sqrt{Q} < m < 1)$

$$E_{1y} = E_0 \sinh \left(\nu \sqrt{m^2 - Q} \frac{x}{a} \right) \quad (21)$$

$$E_{2y} = E_0 \left[\sinh \left(\nu \xi \sqrt{m^2 - Q} \right) \cos \left(\nu \sqrt{1 - m^2} \left(\xi - \frac{x}{a} \right) \right) \right. \\ \left. - \sqrt{\frac{m^2 - Q}{1 - m^2}} \cosh \left(\nu \xi \sqrt{m^2 - Q} \right) \right. \\ \left. \cdot \sin \left(\nu \sqrt{1 - m^2} \left(\xi - \frac{x}{a} \right) \right) \right] \quad (22)$$

$$E_{3y} = - E_0 \left[\cos \left(2\nu \sqrt{1 - m^2} \right) - \sqrt{\frac{1 - m^2}{m^2 - Q}} \right. \\ \left. \cdot \tanh \left(\nu \xi \sqrt{m^2 - Q} \right) \sin \left(2\nu \sqrt{1 - m^2} \right) \right] \\ \cdot \sinh \left(\nu \sqrt{m^2 - Q} \left(2\xi + 2 - \frac{x}{a} \right) \right) \quad (23)$$

and

$$H_x = - \frac{1}{Z_0} E_y \quad (24)$$

where m can be determined from the characteristic equation

$$(m^2 - Q) \cosh^2(\xi \nu \sqrt{m^2 - Q}) \sin(2\nu \sqrt{1 - m^2}) - (1 - m^2) \cdot \sinh^2(\xi \nu \sqrt{m^2 - Q}) \sin(2\nu \sqrt{1 - m^2}) + 2\sqrt{(m^2 - Q)(1 - m^2)} \cdot \cosh(\xi \nu \sqrt{m^2 - Q}) \cdot \sinh(\xi \nu \sqrt{m^2 - Q}) \cos(2\nu \sqrt{1 - m^2}) = 0. \quad (25)$$

Cutoff Frequencies

The cutoff frequencies can be determined from (15) by putting

$$k = 0, \quad \text{i.e., } m = 0$$

$$Q \cos^2(\xi \nu \sqrt{Q}) \sin(2\nu) - \sin^2(\xi \nu \sqrt{Q}) \sin(2\nu) + 2\sqrt{Q} \cos(\xi \nu \sqrt{Q}) \cdot \sin(\xi \nu \sqrt{Q}) \cos(2\nu) = 0. \quad (26)$$

Power Flow in the Structure

From the Poynting theorem, the average power flow through the cross section of the guide is given by

$$P = 1/2 \operatorname{Re} \iint (\bar{E} \times \bar{H}^*) \cdot d\bar{s}$$

where \bar{H}^* is the complex conjugate of the magnetic field vector. Using (14), (19), and (24)

$$P = -1/2 \operatorname{Re} \int_0^b \int_0^{2d} E_y H_x^* dx dy = \frac{b}{2Z_0} \int_0^{2d} |E_y|^2 dx.$$

Since the structure is symmetrical with respect to the plane $x=d$, the power expression can be reduced to

$$P = \frac{b}{Z_0} \int_0^d |E_y|^2 dx. \quad (27)$$

Substituting the values of E_y from the above equations for the different regions, and evaluating the integral, the following expression results:

$$P = \frac{A |E_0|^2}{4(\xi + 1)Z_0} (\rho_1 + \rho_2) \quad (28)$$

where

- A the cross sectional area of the guide,
- Z_0 the wave impedance $= -E_y/H_x$,
- ρ_2 corresponds to the power flow in the electrooptic medium,
- ρ_1 corresponds to the power flow in the remaining part of the wave guide.

The expressions for ρ_1 and ρ_2 for different cases are as follows.

Case a:

$$\rho_1 = \xi - \frac{\sin(2\nu\xi\sqrt{Q - m^2})}{2\nu\sqrt{Q - m^2}} \quad (29)$$

$$\begin{aligned} \rho_2 = & \frac{Q - m^2}{1 - m^2} \cos^2(\nu\xi\sqrt{Q - m^2}) + \sin^2(\nu\xi\sqrt{Q - m^2}) \\ & + \frac{\sqrt{Q - m^2}}{\nu(1 - m^2)} \cos(\nu\xi\sqrt{Q - m^2}) \\ & \cdot \sin(\nu\xi\sqrt{Q - m^2}). \end{aligned} \quad (30)$$

Case b:

$$\rho_1 = \frac{2}{3}\xi^3 \quad (31)$$

$$\rho_2 = \frac{1}{\nu^2(1 - Q)} [1 + \xi + \nu^2\xi^2(1 - Q)]. \quad (32)$$

Case c:

$$\rho_1 = \frac{\sinh(2\nu\xi\sqrt{m^2 - Q})}{2\nu\sqrt{m^2 - Q}} - \xi \quad (33)$$

$$\begin{aligned} \rho_2 = & \frac{m^2 - Q}{1 - m^2} \cosh^2(\nu\xi\sqrt{m^2 - Q}) + \sinh^2(\nu\xi\sqrt{m^2 - Q}) \\ & + \frac{\sqrt{m^2 - Q}}{\nu(1 - m^2)} \cosh(\nu\xi\sqrt{m^2 - Q}) \\ & \cdot \sin(\nu\xi\sqrt{m^2 - Q}). \end{aligned} \quad (34)$$

It is desirable to choose the parameters which maximize the ratio $\rho_2/(\rho_1 + \rho_2)$ since this means more efficient utilization of the incident power.

Dielectric Losses

The attenuation constant α due to the dielectric losses is given by

$$\alpha = \frac{P_L}{2P}$$

where P_L is the power lost in the dielectric and electro-optic media per unit length, and is given by

$$\begin{aligned} P_L = & \int_0^b \int_0^{2d} \frac{|E_y| |J_y|}{2} dx dy \\ = & \frac{b\epsilon''\omega\epsilon_0}{2} \int_0^{2d} |E_y|^2 dx \\ = & \frac{A |E_0|^2 \nu}{4(\xi + 1)amZ_0} [Q\rho_1 \tan \delta_1 + \rho_2 \tan \delta_2] \end{aligned} \quad (35)$$

where $E = \epsilon' - j\epsilon''$ and $\tan \delta = \epsilon''/\epsilon'$.

Then the attenuation constant α will be given by

$$\alpha = \frac{\nu [Q\rho_1 \tan \delta_1 + \rho_2 \tan \delta_2]}{2am(\rho_1 + \rho_2)}. \quad (36)$$

Copper losses in the guide walls are neglected, since they are small except near cutoff where they may be comparable to the dielectric loss. The above expressions will be used in the following section when the modulator design is considered.

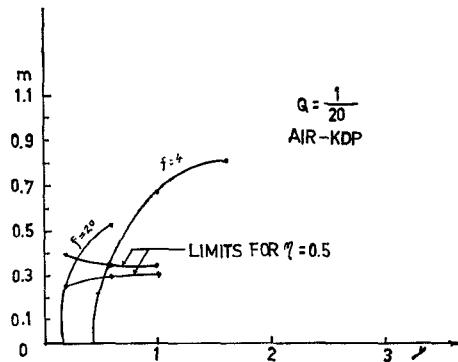


Fig. 2. Normalized characteristic curve, air-KDP.

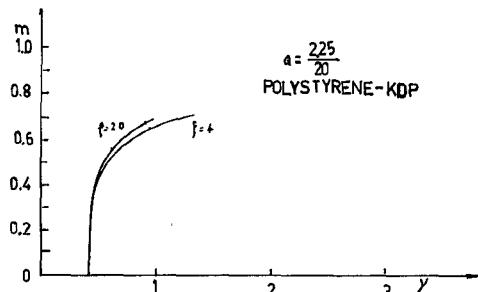


Fig. 3. Normalized characteristic curve, polystyrene-KDP.

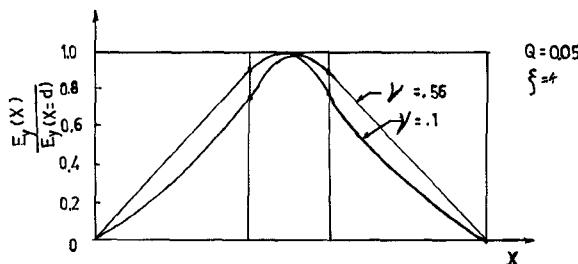


Fig. 4. Electric field variation.

The characteristic equations are solved numerically. Fig. 2 is the normalized characteristic curve which shows the variation of m with ν between the cutoff values of the first and second modes for $Q=1/20$ (air-KDP) $\xi=4$, and $\xi=20$. When air is replaced by polystyrene ($Q=2.25/20$) the normalized characteristic curves are given in Fig. 3. Fig. 4 shows the field variation across the waveguide cross section for $Q=1/20$, $\xi=4$, $\nu=0.56$, and $\nu=0.1$.

IV. MODULATOR DESIGN

When using the structure as a laser modulator, the following requirements should be taken into consideration.

1) The RF modulating signal and the laser beam should propagate in synchronism. This requirement fixes the normalized parameter m to a value given by

$$n = m\sqrt{\epsilon_2}. \quad (37)$$

TABLE I

Case	m	Q	ξ	ν
1)	0.328	0.05	4	0.56
2)	0.328	0.05	20	0.28
3)	0.328	0.1128	4	0.47
4)	0.328	0.1128	20	0.47

For a given EO medium in the waveguide, (37) gives the required value of m .

2) The field variation in the EO crystal should be as small as possible to give uniform modulation across the cross section of the light beam.

3) The total input power is not used for modulation, but only a fraction proportional to p_2 . Therefore, the ratio $p_2/(p_1+p_2)$ should be as large as possible to increase the efficiency.

The design of a laser phase modulator proceeds as follows.

1) Choose a particular EO crystal (to be specific we will choose KDP, and the rest of the discussion will be concerned with this particular crystal) and use (37) to obtain the required value of m which ensures beam-wave synchronization. The parameters for KDP are [2], [3]: $n=1.47$, $\epsilon_2=20$, $r_{63}=9.7 \times 10^{-12} \text{ m/V}$, and $\tan \delta_2=7.5 \times 10^{-3}$. Therefore

$$\frac{n}{\sqrt{\epsilon_2}} = 0.328 = m.$$

2) Use Figs. 2 or 3 to obtain the corresponding value of ν . The parameters obtained are shown in Table I.

The values of ν for polystyrene-KDP (cases 3 and 4) are close to their corresponding cutoff values ($\nu=0.43$). Therefore polystyrene-KDP will be disregarded. The values of m and Q for air-KDP (cases 1 and 2) satisfy the relation $\sqrt{Q} < m < 1$, and therefore, the expressions for case c in section III will be used in the following calculations.

3) Choose a crystal width a ; a practical value for the KDP crystal may be taken as 1 mm. Use (36), (33), and (34) to calculate the attenuation constant α .

4) Choose a modulator length L and use (3) to obtain the amplitude of the electric field E_s' for a specified $\Delta\phi$ and laser wavelength λ_0 .

5) The value of E_s' obtained from step 4) is equated to the right-hand side of (22) from which E_0 is obtained. Equation (22) will be evaluated at the middle of the guide cross section $x=d$ (as will be shown, the variation of the field across the crystal width a is small).

6) Equations (28), (33), and (34) give the required RF modulating power for a chosen cross-sectional area of the guide.

7) The bandwidth of the modulator is determined to a good approximation by the reduction factor η [2], where η is given by

TABLE II

m	Q	ξ	ν	α	P (W)	BW (%)	$E_y(x=d-a)$	p_2
							$E_y(x=d)$	(p_1+p_2)
0.328	0.05	4	0.56	3.04	12	7	0.89	0.473
0.328	0.05	20	0.28	0.544	26	40	0.96	0.17

$$\eta = \frac{\sin u}{u} . \quad (38)$$

At synchronism η equals unity; when there is a velocity difference between the RF modulating wave and the light beam, η will be less than unity. Using the normalized variables previously defined, (38) can be written as

$$m = \frac{n}{\sqrt{\epsilon_2}} \pm \frac{2a}{\nu L} \sqrt{6(1-\eta)} \quad (39)$$

where it has been assumed that u is small such that $\sin u$ can be approximated by the first two terms in its power series expansion. Taking $\eta = 0.5$ as a lower limit and choosing $L = 25$ cm, (39) gives

$$m = 0.328 \pm \frac{0.0152}{\nu} . \quad (40)$$

This equation is also plotted in Fig. 2. The bandwidth is calculated from the intersection of the characteristic curve and the curve $\eta = 0.5$.

8) The value of the electric field at the edge of the EO crystal ($x = d-a$) relative to its value at the center ($x=d$) is calculated to check the modulation uniformity across the beam cross section. Values close to unity give uniform modulation. Also, the ratio $p_2/(p_1+p_2)$ is calculated. A large ratio gives larger efficiency.

Table II shows the results obtained when the above procedure is applied to design a modulator with the following specifications: $a = 0.1$ cm, $b = 0.5$ cm, $L = 25$ cm, air-KDP, $\lambda_0 = 0.69 \mu$, and $\Delta\phi = 0.5$ rad.

These results indicate that the bandwidth and the field uniformity across the beam increases with the in-

crease of ξ , while the opposite is true for the efficiency. The choice between these two cases is decided by the relative importance of the results, e.g., BW, power, etc., in the particular application of the modulator.

V. CONCLUSION

The analysis presented in this paper shows that rectangular waveguides partially loaded with EO crystals can be used as a traveling-wave laser phase modulator. A step-by-step design procedure of the modulator is given from which the RF power, bandwidth, and the efficiency for a specified phase modulation $\Delta\phi$ can be determined.

The requirement of synchronization between the RF modulating signal and the laser beam limits the range of the operating frequency of the RF signal. The numerical example taken, for KDP, indicates that the BW and the field uniformity across the beam increase with the increase of ξ , while the opposite is true for the efficiency (p_2/p_1+p_2) .

ACKNOWLEDGMENT

The authors wish to thank Mrs. Nadia El-Araby of the National Research Center for her assistance in carrying out some of the numerical calculations.

REFERENCES

- [1] I. P. Kaminow and E. H. Turner, "Electrooptic light modulators," *Proc. IEEE*, vol. 54, Oct. 1966, pp. 1374-1390.
E. I. Gordon, "A review of acoustooptical deflection and modulation devices," *Proc. IEEE*, vol. 54, Oct. 1966, pp. 1391-1401.
- [2] I. P. Kaminow and J. Liu, "Propagation characteristics of partially loaded two-conductor transmission line for broadband light modulators," *Proc. IEEE*, vol. 51, Jan. 1963, pp. 132-136.
- [3] C. J. Peters, "Gigacycle bandwidth coherent light traveling-wave phase modulator," *Proc. IEEE*, vol. 51, Jan. 1963, pp. 147-153.
- [4] D. Chen and T. C. Lee, "Propagation characteristics of a partially filled cylindrical waveguide for light beam modulation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, Oct. 1966, pp. 482-486.
- [5] J. L. Putz, "A wide-band microwave light modulator," *IEEE Trans. Electron Devices*, vol. ED-15, Oct. 1968, pp. 695-698.
- [6] P. H. Vartanian, W. P. Ayres, and A. L. Helgesson, "Propagation in dielectric slab loaded rectangular waveguide," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, Apr. 1958, pp. 215-222.
- [7] M. Dore, "A low drive-power light modulator using a readily available material ADP," *IEEE J. Quantum Electron.*, vol. QE-3, Nov. 1967, pp. 555-560.